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Estimation of Layerwise Elastic Parameters of Stiffened Composite Plates

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For the first time, an investigation of stiffened and unstiffened multilayered composite plates has been conducted to determine the material property parameters of the plate as well as the stiffener from numerically simulated experimental modal data and finite element predictions, using model updating techniques. The problem is formulated as a global minimization of the error function, defined by the difference in undamped eigenvalues as well as mode shapes, as predicted from the finite element modeling compared to that obtained experimentally. The parameter estimation problem is solved using an iterative gradient-based minimization algorithm, which can start from any randomly selected set of initial parameters. The position, physical properties, and orientation of stiffeners create considerable variations in the modal properties as compared to the bare plates of similar construction. This makes each of the stiffened plate problems rather unique. A few simulated examples are presented, and the methodology is found to be robust, even in the presence of random noise.

Nomenclature

a	= amplitude of noise
bound$l(i)$	= lower bound of the variable
bound$u(i)$	= upper bound of the variable
E	= Young's modulus of a lamina
E_T	= total error term in the objective function
$E1$	= Young's modulus of a lamina of the plate in the longitudinal material direction
$E2$	= Young's modulus of a lamina of the plate in transverse material direction
$ES1$	= Young's modulus of the lamina of the stiffener in longitudinal material direction
$ES2$	= Young's modulus of the lamina of the stiffener in transverse material direction
E_ω, E_ϕ, E_p	= error terms for frequency, mode shape, and internal penalty, respectively
$G12$, $G13$, $G23$	= shear modulus of the lamina of the plate
$GS12$	= in-plane shear modulus of the lamina of the stiffener
M	= number of measured modes
$n$cons	= number of parameters with constraints
$n$used	= number of modes considered
P_i^L	= lower bound of the ith parameter
P_i^U	= upper bound of the ith parameter
P_i^*	= value of the ith parameter
$PR12$	= in-plane Poisson's ratio of the lamina of the plate
$PS12$	= in-plane Poisson's ratio of the lamina of the stiffener
r	= uniformly distributed sequence of random numbers between -1 and $+1$
$rr(i)$	= constant for evaluating internal penalty
W_ω, W_ϕ, W_p	= weighting factors of frequency, mode shape, and internal penalty, respectively
X	= noise-free reference data
X^*	= noisy data (frequency or component of eigenvector)
$x(i)$	= ith parameter
η	= deterministic design allowable
ρ	= mass density
ϕ	= mode shape
ω	= frequency

Subscripts

a	= analytical
c	= chopped strand mat
j	= jth natural frequency
m	= measured
w	= woven roving

Introduction

STIFFENED composite plates are extensively used in high-performance structures, such as aircraft, missiles, surface effect ships, submersibles, etc., where weight saving is a major criterion without reduction of the stiffness. Because of the typicality in construction of composites and also because of the differences in various manufacturing and curing processes, the material property parameters may differ significantly from those specified by the manufacturers or obtained from established standards. Unlike isotropic materials, experimental characterization of composite materials is much more complex, requiring a large number of property values to be determined. They do not permit determination of all of the basic parameters from a single test. Thus there can be little doubt about the value of an efficient methodology to replace the existing procedures for the determination of material property parameters of composite structures. It would be best if such a structure can be tested as a whole and their actual material properties verified in situ so that all subsequent analysis will be much more realistic.

Correction of finite element models by processing dynamic test data obtained from experimental modal testing[1] is most appropriately described under the heading of model updating and has been an active area of research for nearly three decades.[2,3] The theoretical knowledge bases, as well as the experimental constraints, are application specific and may differ considerably from one field of application to the other. The methodologies differ mainly in how the plate is approximately modeled, the computational aspects of the algorithms used, and the choice of parameters. Dewilde et al.[4,5] used a model developed by Galerkin based on Lagrangian polynomials as assumed shape functions to extract the six elastic rigidities of rectangular anisotropic plates using a least-square technique. Deobald and Gibson[6] used the Rayleigh–Ritz technique to model the vibrations of rectangular orthotropic plates to estimate the elastic constants. Larsson[7] presented an iterative method to determine all four in-plane elastic constants of an oriented strand board (OSB) from a single test method. Dascotte[8] applied the inverse eigensensitivity method

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using only the eigenfrequencies to determine the elastic constants of vertically stiffened composite cylindrical shell. Almost all of the references use only frequency information. Fallstrom and Johnson[9] were the first to attempt using mode shapes along with the measured frequencies. Grediac and Paris[10] also use the natural frequencies and mode shapes to determine the stiffness of anisotropic plates using a direct approach. Several other references also exist[11,12] that address the same problem. In all cases (except Ref. 8) the method is applied to either flat plate or beam specimens in a closely controlled laboratory environment. Application to stiffened composite plates is nonexistent.

The use of the stiffener causes considerable changes to the inplane stiffness properties of a stiffened structure. In this paper for the first time a nondestructive evaluation method is presented to determine the globally averaged in-plane elastic parameters of the plate, as well as the stiffener separately. The method is based on the minimization of the sum of the differences of the modal properties, that is, eigenfrequencies and mode shapes, between the predicted values from the finite element analysis and those determined from the numerical experiment. In this paper, the word experiment means numerically simulated experiments only. An attempt has also been made, for the first time, to estimate the layerwise elastic constants of a laminate from a single test. Particular attention is given to the effect of random noise present in experimental data on the estimated parameters.

Estimation of Parameters

The procedure for identifying physical parameters consists of two separate approaches: the forward problem and the inverse problem. The forward problem consists of numerical simulation of experimental modal databases and analytical predictions of modal databases using an accurate finite element model. The inverse problem consists of the formation of the objective function and an efficient solution of the problem by a minimization algorithm. The algorithm switches over from one to the other during the iterative process and is shown in Fig. 1.

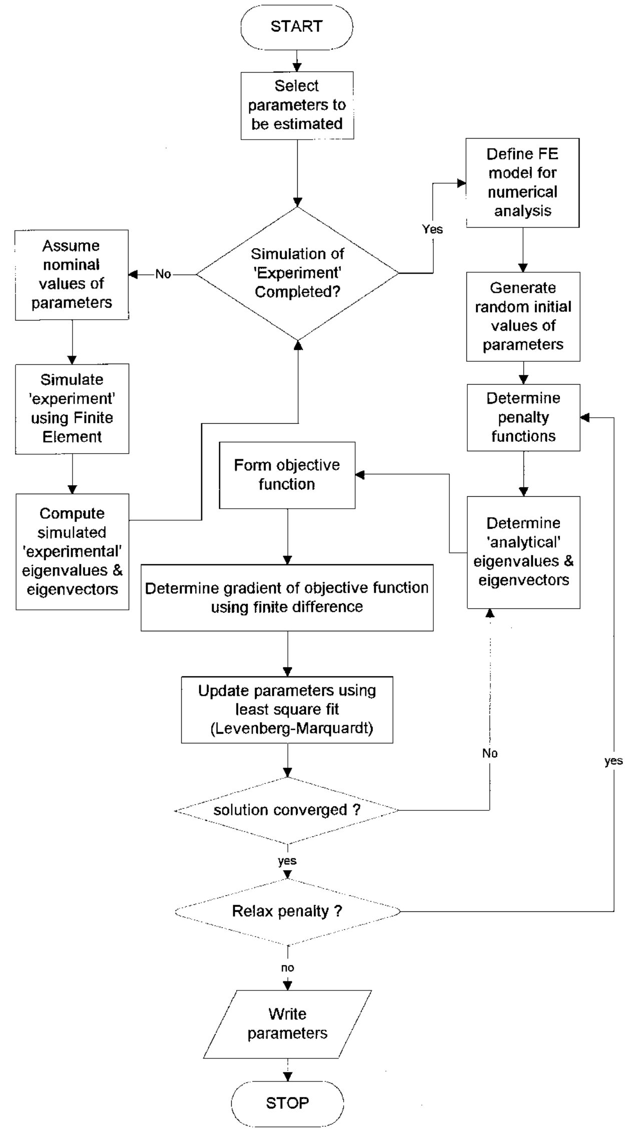

Fig. 1 Flowchart of the package.

Forward Problem

Numerical Simulation of Experimental Data

The finite element technique is used to generate the necessary numerical experimental modal database, assuming that our hypothetical real system is discrete. A stiffened plate bending element, details of which are given later, has been used.[13–15] A set of reference material constants is realistically chosen. The geometry and mass density are assumed to be accurately determined, and the boundary conditions are assumed to act perfectly. Furthermore, only the first few natural frequencies, along with some selected coordinates of the corresponding mode shapes, are assumed to be measured, and to coincide with the finite element nodes of the theoretical model. Noisy data sets are generated by adding uniformly distributed random noise to the noise-free experimental reference data sets as follows:

$$X^* = X(1 + ra) \tag{1}$$

Using a finite element for the simulation of an experiment is a bit artificial, but it facilitates the study of a particular feature, without being distracted by certain true information, such as damping, noise, etc., as is always present in real experimentation.

Finite Element Modeling

The isoparametric quadratic plate bending element[13,14] and a three-noded beam element[15] have been chosen to model the stiffened plate, as used in simulating the experimental results. The element can incorporate transverse shear deformation through a first-order approximation. Only a rectangular stiffener is considered here. The stiffener, as well as the plate, may consist of several layers having different material properties.[16]

A simultaneous iteration algorithm has been implemented to calculate the free undamped modal properties of the structure.[17]

Inverse Problem

Formation of Objective Functions

The objective function consists of three terms: a term relating to the error in natural frequencies E_ω, a term relating to the error in mode shapes E_ϕ, and another term for the internal penalty that "explodes" at the boundary[18] E_p. These individual terms have relative weights. The total error term is

$$E_T = W_\omega E_\omega + W_\phi E_\phi + W_p E_p \tag{2}$$

The objective function for the frequency error is just a weighted sum of square of the differences in the natural frequencies, provided the modes are paired correctly, which can be checked by modal assurance criteria (MAC)[1] at each iteration. The MAC value is 1 for perfect correlation,

$$E_\omega = \sum_{j=1}^{n\text{used}} W_{\omega_j} \left(\omega_{m_j} - \omega_{a_j}\right)^2 \tag{3}$$

The mode shape error function is

$$E_\phi = \sum_{j=1}^{n\text{used}} W_{\phi_j} \left(\phi_{m_j} - \phi_{a_j}\right)^T \left(\phi_{m_j} - \phi_{a_j}\right) \tag{4}$$

Only a limited number of degrees of freedom chosen from the analytical mode shapes are paired with the corresponding measured set of coordinates.

An internal penalty is imposed indirectly[18] by augmenting the penalty term to the objective function:

$$E_p = \sum_{i=1}^{ncons} rr(i)\left[\frac{1.0}{\text{bound}u(i) - x(i)} + \frac{1.0}{x(i) - \text{bound}l(i)}\right] \quad (5)$$

The value of $rr(i)$ is reduced in subsequent stages, as the convergence improves.

A frequent source of trouble is the disparity of weight between the various functions, whereby one part may overpower the other. Here the orders of the different parts of the objective functions are made nearly same to make the system equation well conditioned. However, whenever sufficient frequency information is available, they are weighed more than the eigenvectors.

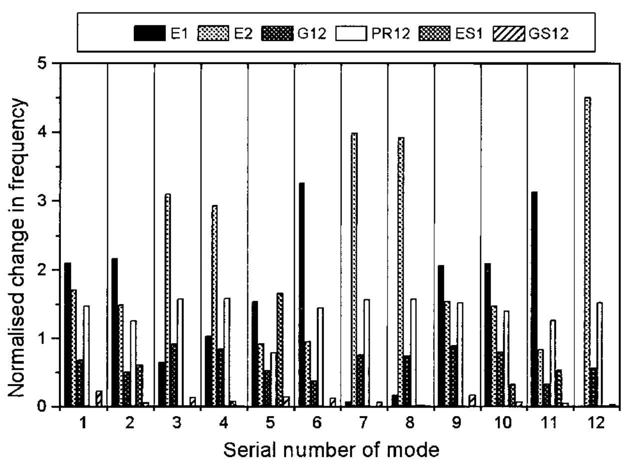

Fig. 2 Frequency sensitivity diagram for problem 1 (single stiffener).

Sensitivity Analysis

A sensitivity analysis is carried out to determine the physical involvement of different parameters. A set of parameters is chosen first as a reference. Then a finite element analysis is carried out to determine the eigenfrequencies for use as another reference set. Then each parameter is increased from its reference by 10% in turn, and a new finite element analysis is carried out. The normalized value of increased frequency is the relative sensitivity of that parameter at that mode.[7] This sensitivity diagram provides a basis to decide the inclusion of a particular set of modes in the system identification algorithm.

Selection of Initial Values

The question of variability of physical parameters of composites is still not resolved, and no definite rule exists.[19] It can be established probabilistically after acquiring a considerable amount of data. Alternatively, a deterministic design allowable can be assumed, and the algorithm can start from a randomly selected parameter set within the upper and lower bounds given by

$$P_i^L = P_i^*(1 - \eta) \quad (6)$$

$$P_i^U = P_i^*(1 + \eta) \quad (7)$$

Minimization Algorithm

Levenberg–Marquardt's algorithm (see Ref. 20), which is basically a gradient-based method for estimation of parameters, is

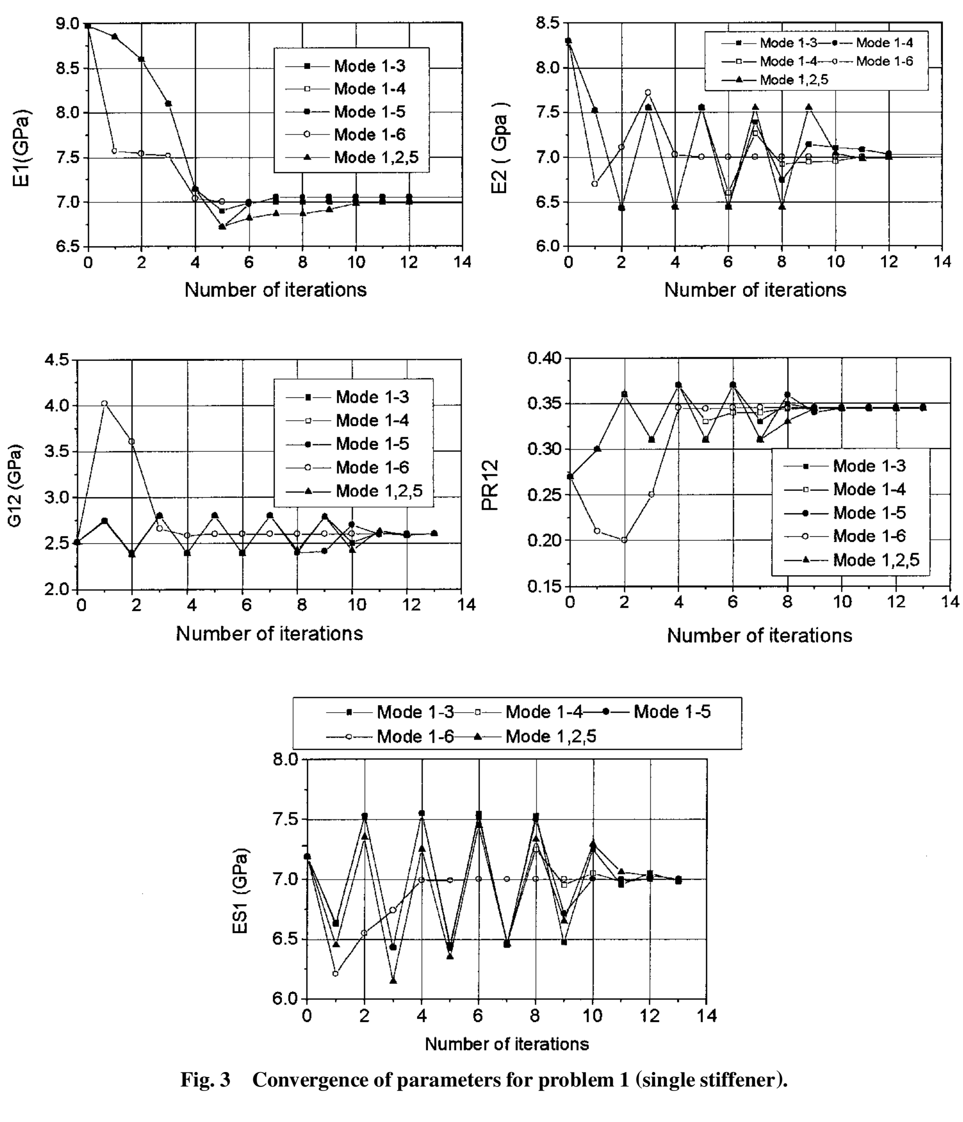

Fig. 3 Convergence of parameters for problem 1 (single stiffener).

implemented here. This method combines both the features of Cauchy's steepest descent method (which works well when the initial point is far away from minimum point) and Newton's method (which works well when the initial guess is nearer[21]). Its choice of a proper step size for minimization is adaptive. The internal penalty is relaxed during the subsequent steps of iteration. A forward difference approximation is made in calculating the derivative of the objective functions with respect to the parameters. If multiple minima are located, the parameter sets having the least norm of the objective function is taken as the global solution. The algorithm can use any predefined number of coordinates and frequencies in the order as defined by the user. For example, into the present investigation, 25 points uniformly distributed over the plate has been considered for the first three plates.

Numerical Examples

Organization of Study

A variety of stiffened plate problems having a varying number of stiffeners has been studied. The modal properties, that is, the frequencies and mode shapes computed using the present code, compare well with standard references.[22–24]

The procedure for each example is repeated for both noise-free and noisy data sets. For each data set, 10 incarnations are taken, that is, the parameter estimation algorithm is run 100 times for each set of the selected experimental modal data sets. However, a typical set of convergence curves from a particular initial parameter set for each problem has been presented here. The algorithm has been tested for different levels of upper and lower bounds; however, the bounds considered in the following examples are calculated using $\eta = \pm 30\%$ for problems 1–3 and $\eta = \pm 50\%$ for problem 4.

Discussion of Results

Problem 1: Uniaxially Stiffened Clamped Composite Plate with a Single Stiffener

The first problem investigated is a plate of size 375 × 300 mm, stiffened by a centrally placed single eccentric stiffener, parallel to the shorter direction of the plate. The stiffener laminations are perpendicular to the plane of the plate.[22] The thickness of the plate is 2.208 mm. The breadth and height of the stiffener are 6 and

Table 1 Influence of random errors on the identified parameters for problem 1 (single stiffener)

	Modes 1–3[a] random error, %		Modes 1–4[a] random error, %		Modes 1–5[a] random error, %		Modes 1–6[a] random error, %		Modes 1, 2, 5[a] random error, %	
Parameters	5	10	5	10	5	10	5	10	5	10
$E1$, GPa										
Mean	7.0	7.02	6.99	6.99	7.05	7.05	6.99	6.99	6.87	6.86
SD	0.07	0.07	——	——	——	——	——	——	——	——
% Error	——	——	——	——	0.7	0.7	——	——	1.8	2.0
$E2$, GPa										
Mean	7.03	7.1	7.03	7.03	7.01	7.01	7.11	7.11	6.91	6.9
SD	0.07	0.1	——	——	——	——	0.03	0.03	——	——
% Error	——	——	0.4	0.4	0.14	0.14	1.5	1.5	1.4	1.4
$G12$, GPa										
Mean	2.6	2.59	2.6	2.6	2.6	2.6	2.78	2.78	2.61	2.61
SD	——	——	——	——	——	——	——	——	0.02	0.02
% Error	——	——	——	——	——	——	6.9	6.9	0.4	0.4
$PR12$										
Mean	0.344	0.343	0.344	0.344	0.344	0.344	0.344	0.344	0.345	
SD	——	0.03	——	——	——	——	——	——	——	
% Error	——	0.5	——	——	——	——	——	——	——	
$ES1$, GPa										
Mean	6.9	6.9	6.98	6.98	6.99	6.99	6.48	6.48	7.04	6.96
SD	0.07	0.16	——	——	——	——	——	——	0.13	0.4
% Error	1.7	1.7	0.2	0.2	0.14	0.14	7.0	7.0	0.51	0.57

[a]Selected mode combinations.

Table 2 Influence of random errors on the identified parameters for problem 2 (cross stiffener)

	Modes 1–3[a] random error, %		Modes 1–4[a] random error, %		Modes 1–5[a] random error, %		Modes 1–6[a] random error, %		Modes 1, 2, 4, 7[a] random error, %	
Parameters	5	10	5	10	5	10	5	10	5	10
$E1$, GPa										
Mean	144.72	141.10	145.63	145.89	145.51	145.52	146.73	147.17	144.91	144.89
SD	3.86	4.1	2.25	2.22	0.96	0.95	2.62	3.4	1.37	0.8
% Error	0.05	2.5	0.57	0.75	0.49	0.49	1.33	1.63	0.07	0.07
$E2$, GPa										
Mean	9.62	9.62	9.63	9.62	9.63	9.62	9.66	9.63	9.66	9.66
SD	——	——	0.01	——	——	——	——	——	——	——
% Error	0.52	0.6	0.4	0.5	0.4	0.5	0.1	0.4	0.1	0.1
$G12$, GPa										
Mean	4.13	4.13	4.15	4.13	4.14	4.14	4.12	4.12	4.13	4.11
SD	——	——	——	——	——	——	——	——	——	——
% Error	0.24	0.25	0.2	0.24	——	——	0.48	0.48	0.24	0.72
$ES1$, GPa										
Mean	155.5	155.73	153.25	153.3	154.25	154.25	152.97	155.89	146.87	144.4
SD	0.3	0.3	2.3	2.1	2.2	2.2	2.6	2.1	2.31	0.87
% Error	7.0	7.5	5.83	5.87	6.52	6.52	5.64	7.65	1.42	0.27
$ES2$, GPa										
Mean	8.59	8.59	8.62	8.61	8.67	8.67	9.21	8.65	9.24	9.18
SD	——	——	——	——	——	——	0.4	——	0.05	——
% Error	11.1	11.1	10.8	10.8	10.65	10.65	4.75	10.54	4.4	5.06

[a]Selected mode combinations.

12 mm, respectively. The typical material parameters are $E1 = E2 = 7$ GPa, $G12 = G13 = G23 = 2.6$ GPa, $PR12 = 0.345$, $\rho = 1504.2$ kg/m^2. There are two layers for both the plate and the stiffener. The stacking sequences were symmetric (0/0).

The present investigation includes the first six modes only, with 12×12 mesh divisions. It is found that the sensitivities of the frequencies with respect to $ES2$ and $PS12$ are negligible; therefore, the present choice of objective function is not suitable for their identification. The parameters chosen for identification are $E1$, $E2$, $G12$, $PR12$, and $ES1$, and the sensitivities of the frequencies with respect to them are shown in a typical frequency sensitivity diagram in Fig. 2.

As explained earlier, relevant choice of mode is essential for identification. Here, from Fig. 2, it is clear that modes 1, 2, and 6 are most affected by $E1$, whereas modes 3 and 4 are most affected by $E2$ and $G12$. For $ES1$, modes 2 and 5 are most affected. For $PR12$ no such preferences exist.

Convergence characteristics from a typical random set of initial values are shown in Fig. 3. As expected, the best convergence for all of the variables is obtained when all of the six modes are included. $E1$ has converged well when at least the first two modes are included. No considerable difference exists in the convergence characteristics of $PR12$, as expected. Other parameters also indicated good convergence.

Table 3 Influence of random errors on the identified parameters for problem 3 (multiple stiffener)

	Modes 1–3[a] random error, %		Modes 1–4[a] random error, %		Modes 1–5[a] random error, %		Modes 1–6[a] random error, %		Modes 1, 2, 4, 5[a] random error, %	
Parameters	5	10	5	10	5	10	5	10	5	10
$E1$, GPa										
Mean	9.16	9.05	9.67	9.7	9.7	9.7	9.7	9.7	9.68	9.65
SD	0.4	0.8	0.08	0.02	——	——	——	——	0.11	0.21
% Error	5.6	6.8	0.4	0.1	0.1	0.1	0.1	0.1	0.3	0.6
$E2$, GPa										
Mean	3.16	3.08	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.23
SD	0.1	0.2	0.1	0.1	——	——	——	——	0.01	0.02
% Error	2.77	5.2	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.6
$G12$, GPa										
Mean	0.9024	0.9024	0.9024	0.9024	0.9025	0.9025	0.9025	0.9025	0.9023	0.9029
SD	——	——	——	——	——	——	——	——	——	——
% Error	——	——	——	——	——	——	——	——	——	0.04
$PR12$										
Mean	0.287	0.287	0.289	0.289	0.285	0.285	0.289	0.289	0.289	0.289
SD	0.01	0.03	——	——	——	——	——	——	——	——
% Error	1.0	1.0	——	——	1.7	1.7	——	——	——	——
$ES1$, GPa										
Mean	10.52	10.52	9.72	9.70	9.72	9.70	9.70	9.70	9.77	9.81
SD	0.84	1.43	0.1	——	0.01	——	——	——	0.23	0.41
% Error	8.34	8.34	0.1	0.1	0.1	0.1	0.1	0.1	0.61	1.02
$ES2$, GPa										
Mean	3.28	3.32	3.24	3.24	3.24	3.24	3.24	3.24	3.24	3.248
SD	0.05	0.14	——	——	——	——	——	——	——	——
% Error	0.92	2.1	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4

[a]Selected mode combinations.

Table 4 Influence of random errors on the identified parameters for problem 4 (unstiffened)

	Modes 1, 2, 4[a] random error, %		Modes 1–4[a] random error, %		Modes 1–6[a] random error, %	
Parameters	5	10	5	10	5	10
E_w, GPa						
Mean	15.1	15.13	14.31	14.67	14.09	14.53
SD	0.16	0.07	0.44	0.89	1.32	0.96
% Error	0.67	0.86	4.6	4.6	6.0	3.13
$G12_w$, GPa						
Mean	3.47	3.47	3.44	3.44	3.46	3.47
SD	0.04	0.03	0.09	0.08	0.03	0.02
% Error	0.85	0.85	1.7	1.7	1.1	0.85
$PR12_w$						
Mean	0.168	0.168	0.165	0.162	0.162	0.165
SD	——	——	——	——	——	——
% Error	1.2	1.2	2.9	4.7	4.7	2.9
E_c, GPa						
Mean	7.98	7.89	7.76	7.45	7.54	7.5
SD	0.02	0.15	0.16	0.45	0.34	0.62
% Error	0.25	1.37	3.0	6.8	5.75	6.25
$G12_c$, GPa						
Mean	2.99	2.98	2.96	2.98	2.92	2.99
SD	0.02	——	0.06	0.01	0.06	0.01
% Error	0.33	0.67	1.33	0.66	2.6	0.33
$PR12_c$						
Mean	0.269	0.263	0.295	0.297	0.295	0.29
SD	0.02	0.04	——	——	——	——
% Error	11.1	10.0	0.33	1.0	0.33	2.07

[a]Selected mode combinations.

Because it is necessary to estimate the effect of errors in the eigenvalues and eigenvectors to the estimated parameters, 1% random error in measured eigenvalues and 5 and 10% errors in measured eigenvectors are introduced. The mean values, standard deviations (SD), and percentage error are computed and are presented in Table 1. The results for noisy data are seen to be independent of the starting point and depend only on the level of noise, as has been observed by other investigators.[10] The convergence characteristics remain similar to that of the no-noise case. The results shown in Table 1 indicate that the percentage error is negligible.

Problem 2: Cross-Stiffened Clamped Composite Plate with Two Stiffeners

The second problem investigated a centrally cross-stiffened clamped plate 254 × 254 mm. The plate, as well as the stiffener, is made of antisymmetric cross ply (0/90), with two layers. The stiffener is eccentric, and the stiffener lamination is parallel to the plate midplane.[23]

The thickness of the plate is 12.7 mm. The breadth and height of stiffener are 6.35 and 25.4 mm, respectively. The material properties are $E1 = 144$ GPa, $E2 = 9.67$ GPa, $G12 = G13 = 4.14$ GPa, $G23 = 3.45$ GPa, $PR12 = 0.3$, and $\rho = 1387.23$ kg/m^3. Up to seven modes are considered for the present investigation; 12 × 12 mesh divisions are found to be accurate enough for this purpose. The sensitivities of the frequencies with respect to $PR12$, $PS12$, and $GS12$ are found to be negligible. Therefore, the parameters selected are $E1$, $E2$, $G12$, $ES1$, and $ES2$. The convergence characteristics for different mode combinations are shown in Fig. 4. In the presence of random noise, the best combination of modes has been found to be 1, 2, 4, and 7, as indicated in Table 2.

Problem 3: Uniaxially Multistiffened Simply Supported Composite Plate with Three Stiffeners

The third problem investigated is a simply supported plate with three unidirectional stiffeners, each oriented parallel to the shorter axis. The details of the geometrical parameters are given in Fig. 5. The stiffeners, as well as the plate, consisted of three layers each, parallel to the plane of the plate.[24] The stacking sequence is (90/0/90). The material parameters are $E1 = 9.71$ GPa, $E2 = 3.25$ GPa, $G12 = G13 = 0.9025$ GPa, $G23 = 0.2356$ GPa, $PR12 = 0.29$, and $\rho = 1347.0$ kg/m^3. For the present investigation, 12 × 12 mesh divisions are used. Only the first six modes are considered.

The convergence characteristics shown in Fig. 6 indicate that no considerable differences exist between the different choices of modes selected here. Table 3 shows the estimated values of parameters in the presence of noise as before. From the frequency sensitivities, it was found that the relative sensitivities of frequencies with respect to $ES1$ and $ES2$ are less, compared to other parameters. Therefore, they are the most affected in presence of noise. As before, note that $G12$ and $PR12$ are estimated very consistently even in the presence of noise, probably due to the almost equal sensitivities at all modes. However, the error for all of the parameters is maximum when the modal information is sparse.

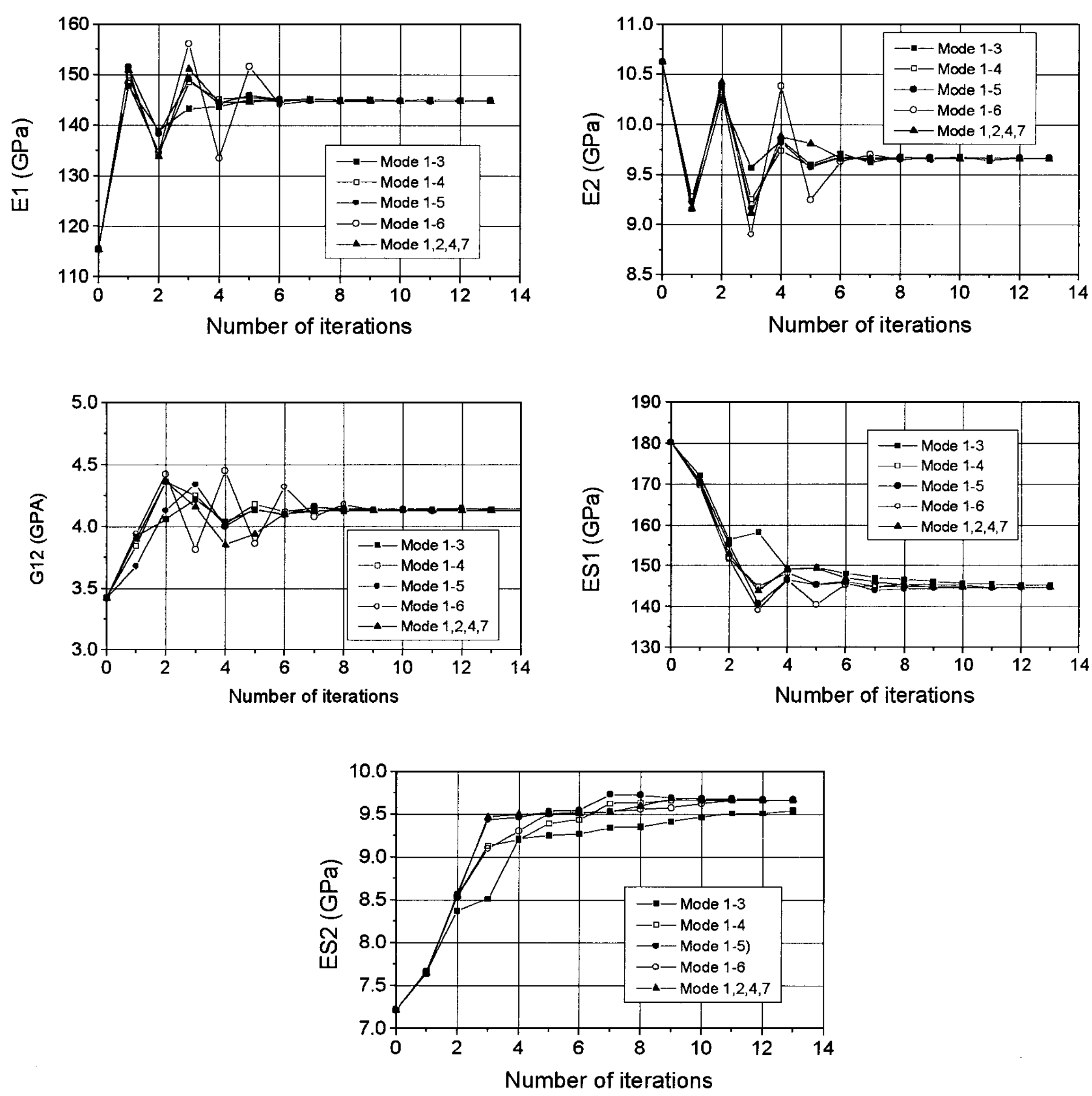

Fig. 4 Convergence of parameters for problem 2 (cross stiffener).

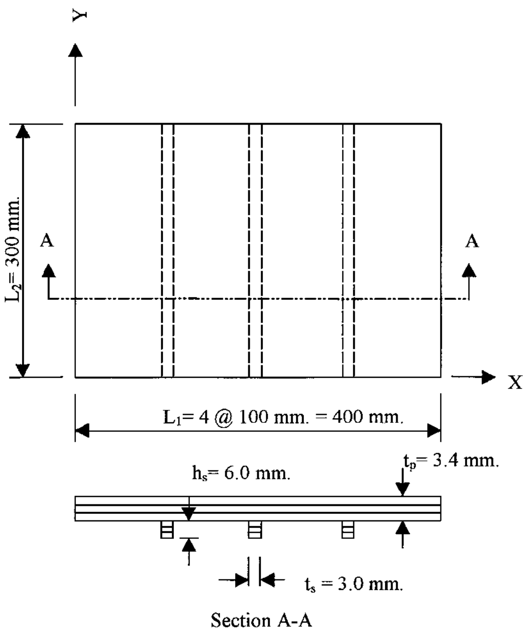

Fig. 5 Uniaxially multistiffened simply supported composite plate.

Problem 4: Unstiffened Multilayer Composite Plate

The fourth problem considers an unstiffened plate having multiple layers and the layer-wise elastic properties are determined.

The composite plate of size 400×300 mm and of thickness 5.0 mm consists of four layers made of two layers of chopped strand mat (CSM) as the inner core, each 1.7 mm thick, and two outside layers made of woven roving (WR), each 0.8 mm thick. Completely free–free boundary conditions are assumed. The present investigation uses 8×8 mesh divisions for the whole plate. The parameters that have been estimated here are E, $G12$, and $PR12$ for both WR and CSM. Thus, there are six parameters. The nominal values of the material parameters are $E_w = 15$ GPa, $G12_w = 3.5$ GPa, $PR12_w = 0.17$, $E_c = 8$ GPa, $G12_c = 3.0$ GPa, and $PR12_c = 0.296$. The layup is (0/0/0/0). Only first six modes are considered for estimation. Eigenvectors were measured at some 16 uniformly distributed locations.

The value of E is best estimated when modes 2, 4, and 6 are included. Similarly, $G12$ value is best estimated with modes 1 and 3. As such, the sensitivity of $PR12$ is less compared to E and $G12$, a better estimate can be obtained using modes 4 and 6.

The problem is solved in more than one step, where the penalty functions are released in subsequent steps. The convergence characteristics in Fig. 7 show convergence into the first stages of iteration. The mode combination 1–2–4 converges before the other combinations. In the presence of noise, all other parameters except $PR12_c$

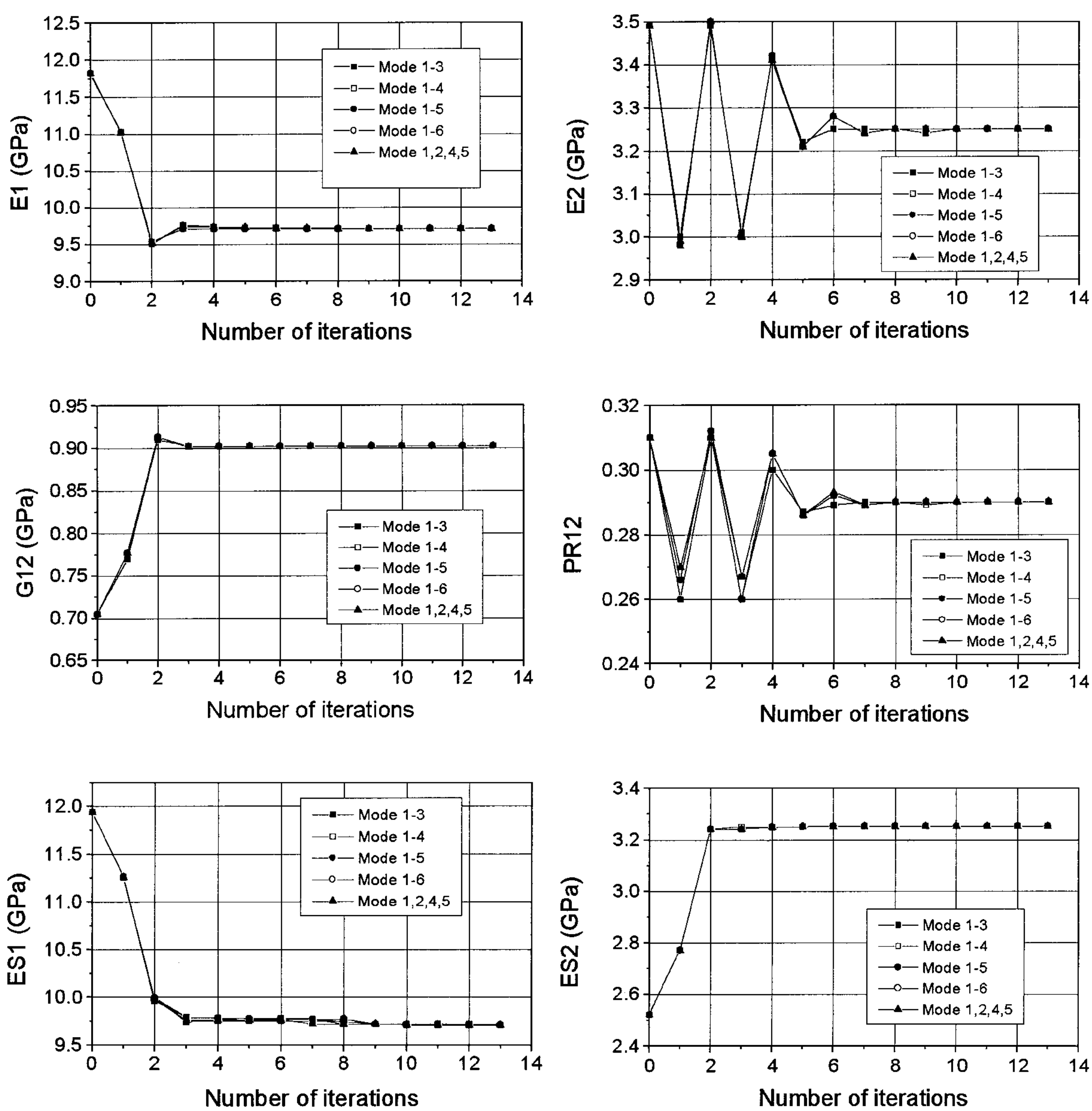

Fig. 6 Convergence of parameters for problem 3 (multiple stiffeners).

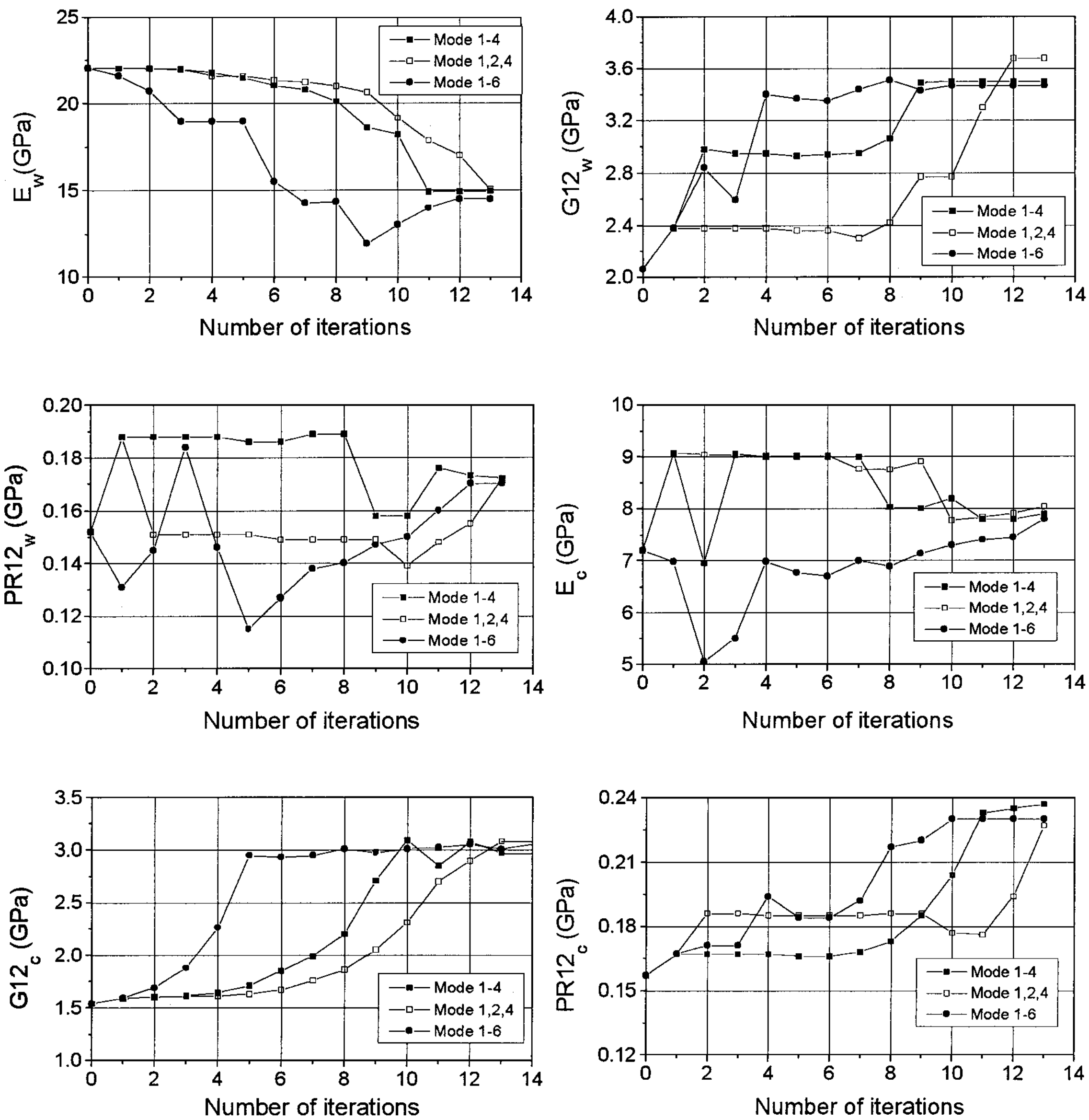

Fig. 7 Convergence of parameters for problem 4 (CSM and WR).

are estimated with most error because the sensitivity of the modal parameters with respect to this parameter is the least. The sensitivity to random noise is shown in Table 4.

Conclusions

An iterative method to determine the in-plane material parameters of composite stiffened and unstiffened plates has been proposed. This is the first time that a stiffened composite plate problem of this nature has been investigated. The boundary conditions are free, simply supported, and clamped. The stiffener lamination can be parallel to the plate, as well as perpendicular to the plate. Although the methodology is computer intensive, it is seen to work well even in presence of random noise. Here the parameters chosen for updating are the variables used in designing a composite plate, thereby making the interpretation of each estimated parameter physically verifiable. Because the eigenvector information is also included, only a few modes are necessary. The proposed method may find application in situ determination of material constants of existing stiffened structures. Layerwise determination of elastic constants has been attempted for a multilayer laminate, and the method is observed to be working within certain bounds over the variable, which can be chosen realistically from practical considerations.

The major contributions of this paper are a thorough investigation of the estimation of material constants for rectangular stiffened plates with different layups and with different boundary conditions and an introduction to the methodology for determining layerwise elastic constants for a multilayer composite plate having differential properties at each layer. The material constants for the plate and the stiffeners are estimated separately. The methodology is found to be stable and precise, but careful judgement is required for selecting the initial parameters and internal penalties.

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